Course Code:	24BSMA101
Course Name:	MATRICES AND CALCULUS

Module-5 – Multiple Integrals

Part - A / 1 Mark/ MCQ				
Sl. No.	Questions	Marks Split-up	K – Level	CO
1.	$\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dydx$= a) 3/2 b) 1/2 c) -1/2 d) -3/2	1	K2	CO5
2.	The value of ∬xye^{x+y} dxdy. a) $ye^y (xe^x - e^x)$ b) $(ye^y-e^y)(xe^x-e^x)$ c) $(ye^y-e^y)xe^x$ d) $(ye^y-e^y)(xe^x+e^x)$	1	K2	CO5
3	The value of integral $\int_0^2 \int_0^x e^{x+y}\, dx\, dy$ is a) $\frac{1}{2}(e-1)$ b) $\frac{1}{2}(e^2-1)^2$ c) $\frac{1}{2}(e^2-e)$ d) $\frac{1}{2}\left(e-\frac{1}{e}\right)^2$	1	K2	CO5
4.	What is the region of $\int_0^1 \int_x^1 dydx$ represents a) Rectangle b) Square c) Circle d) Triangle	1	K2	CO5
5	The area of an ellipse is a) πr^2 b) π c) r^2 d) πr	1	K2	CO5
6	The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the xy plane is a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$	1	K2	CO5
7.	The limits of the integral $\iint_R f(x,y)dydx$ where R is bounded by y = x^2, x = 1 and x-axis. a) Y:0 to x^2, X:0 to 1 a) X:0 to y, Y:0 to 1 c) Y:0 to x^2, X:1 to 2 d) X:0 to y, Y:1 to 2	1	K2	CO5
8	If R is the region bounded x = 0, y = 0, x + y = 1 then $\iint_R dydx$ is equal to a)1/3 b)1/2 c)1 d)1/4	1	K2	CO5
9	The value of $\int_0^3 \int_1^2 xy(x+y)dxdy$ a) 21 b) 22 c) 23 d) 24	1	K2	CO5
10	The value of $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \cos(x+y)dx\, dy$ is	1	K2	CO5

	a) -2 b) 2 c) 0 d) None of these			
11	Transform into polar coordinates the integral $\int_0^a \int_y^a y\, dxdy$ is a) I=$\int_0^{\frac{\pi}{4}} \int_0^{asec\theta} f(r,\theta)\, r\, drd\theta$ b) I=$\int_0^{\frac{\pi}{2}} \int_0^{\infty} r^2\, drd\theta$ c) I=$\int_0^{\frac{\pi}{4}} \int_0^{\infty} sin\theta\, rdrd\theta$ d) I=$\int_0^{\frac{\pi}{2}} \int_0^{1} r^2\, sin\theta\, drd\theta$	1	K2	CO5
12	Transform $\int_0^1 \int_0^{\infty} y\, dxdy$ into polar coordinates a) I=$\int_0^{\frac{\pi}{2}} \int_0^{\infty} r^2\, sin\theta\, drd\theta$ b) I=$\int_0^{\frac{\pi}{2}} \int_0^{\infty} r^2\, drd\theta$ c) I=$\int_0^{\frac{\pi}{2}} \int_0^{\infty} sin\theta\, drd\theta$ d) I=$\int_0^{\frac{\pi}{2}} \int_0^{1} r^2\, sin\theta\, drd\theta$	1	K2	CO5
13	Change the order of integration in $\int_0^1 \int_x^1 dydx$ is a)$\int_0^1 \int_x^1 dydx$ b)$\int_0^1 \int_0^1 dydx$ c)$\int_0^1 \int_x^1 dydx$ d) $\int_0^1 \int_x^1 dxdy$	1	K2	CO5
14	Change the order of integration in $\int_0^a \int_y^a f(x,y)dxdy =$ a) $\int_0^a \int_x^a f(x,y)dydx$ b) $\int_0^a \int_0^x f(x,y)dydx$ c) $\int_0^a \int_a^x f(x,y)dydx$ d) $\int_0^a \int_x^0 f(x,y)dydx$	1	K2	CO5
15	The area bounded between r = 2cosθ and r = 2sinθ is a) π b) 2 π c) 3 π d) 0	1	K2	CO5
16	Area between the parabolas $y^2 = 4x$ and $x^2 = 4y$ is a) $\int_0^2 \int_{x^2/2}^{2\sqrt{x}} dydx$ b) $\int_0^4 \int_{x^2/2}^{2\sqrt{x}} dydx$ c) $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} dydx$ d) $\int_0^4 \int_{x^2/4}^{4\sqrt{x}} dydx$	1	K2	CO5
17	Change of variables from x, y into u, v is $\iint_R f(x,y)dydx = \iint_R F(u,v)\lvert J\rvert dudv$. Here J represents a) $J = \frac{\partial(x,y)}{\partial(u,v)}$ b)$J = \frac{\partial(u,y)}{\partial(x,v)}$ c) $J = \frac{\partial(x,v)}{\partial(u,y)}$ d) $J = 0$	1	K2	CO5
18.	If $u = x + y$ and $v = x - 2y$, then the area element $dxdy$ is replaced by ______ $dudv$. a) 3 b) -3 c) 2 d) -2	1	K2	CO5

19.	The value of $\iint dxdy$ over the rectangle $0 \le x \le 1$ and $0 \le y \le 3$ is ______. a) 1 b) 2 c) 3 d) 4	1	K2	CO5
20.	On changing to polar coordinates $\iint dxdy$ over the circle $x^2 + y^2 = 4$ becomes a) $\int_0^{2\pi}\int_0^2 rdrd\theta$ b) $\int_0^{2\pi}\int_0^4 rdrd\theta$ c) $\int_0^{\pi}\int_0^2 rdrd\theta$ d) $\int_0^{\pi}\int_0^4 rdrd\theta$	1	K2	CO5
21.	The value of $\int_0^1\int_0^2\int_0^3 dxdydz$ is a) 3 b) 6 c) 9 d) 12	1	K2	CO5
22.	The value of the integral $\int_0^a \int_0^a \int_0^a (xy + yz + zx) dx\, dy\, dz$ is a) $\frac{3a^3}{4}$ b) $\frac{2a^5}{3}$ c) $\frac{3a^3}{4}$ d) $\frac{5a^3}{3}$	1	K2	CO5
23.	The value of the integral $\int_0^1 \int_0^2 \int_1^2 x^2yz\, dx\, dy\, dz$ is a)7/3 b)5/3 c)2/3 d)1/3	1	K2	CO5
24.	The value of the integral $\int_0^1 \int_0^2 \int_1^2 xy\, dx\, dy\, dz$ is a) 0 b) 1 c) 2 d) 3	1	K2	CO5
25.	The value of the integral $\int_0^2 \int_1^3 \int_1^2 xy^2z\, dz\, dy\, dx$ is a) 0 b) 16 c) 26 d) 36	1	K2	CO5
26.	The value of the integral $\int_0^1 \int_0^2 \int_0^3 xyz\, dz\, dy\, dx$ is a) 0 b) 1 c) 7/2 d) 9/2	1	K2	CO5
27.	$\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dydxdz$ is equal to a) 4 b) -4 c) 0 d) None of these	1	K2	CO5
28.	$\int_0^1 \int_{y^2}^2 \int_0^{1-x} x\, dzdxdy =$ a) $\frac{2}{35}$ b) $\frac{4}{35}$ c) $\frac{4}{17}$ d) $\frac{2}{17}$	1	K2	CO5
29	Let $I = \int_{x=0}^1 \int_{y=0}^{x^2} x\, y^2\ dy\, dx$, then after change of integration, I may be expressed as a) $\int_{y=0}^1 \int_{x=0}^{\sqrt{y}} x\, y^2\ dx\, dy$ b) $\int_{y=0}^1 \int_{x=\sqrt{y}}^1 y\, x^2\ dx\, dy$ c) $\int_{y=0}^1 \int_{x=\sqrt{y}}^1 x\, y^2\ dx\, dy$ d) $\int_{y=0}^1 \int_{x=0}^{\sqrt{y}} y\, x^2\ dx\, dy$	1	K2	CO5

30-.	To change cartesian (x, y, z) to spherical polar coordinate (r, θ, φ), $dxdydz$ is replaced by ______. a) $r^2 \cos\theta drd\theta d\varphi$ b) $r^2 \sin\theta drd\theta d\varphi$ c) $r\cos\theta drd\theta d\varphi$ d) $r\sin\theta drd\theta d\varphi$	1	K2	CO5

Part - B / 2 Marks				
Sl.No.	Questions	Marks Split-up	K – Level	CO
1.	Evaluate $\int_0^1 \int_1^2 x(x+y)dydx$.	2	K2	CO5
2.	Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$	2	K2	CO5
3.	Evaluate $\int_1^b \int_1^a \frac{dxdy}{xy}$	2	K2	CO5
4.	Evaluate $\int_0^1 \int_0^{\sqrt{x}} xy(x+y)dxdy$	2	K2	CO5
5.	Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$	2	K2	CO5
6	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{sin\theta} rd\theta dr$	2	K2	CO5
7	Sketch roughly the region of integration for $\int_0^1 \int_0^x f(x,y)dydx$	2	K2	CO5
8	Change the order of integration in $\int_0^a \int_0^x f(x,y)dydx$	2	K2	CO5
9	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dydx$	2	K2	CO5
10	Evaluate $\int_1^2 \int_0^y \frac{dxdy}{x^2+y^2}$.	2	K2	CO5
11	Transform into polar coordinates the integral $\int_0^a \int_y^a f(x,y)dxdy$	2	K2	CO5
12	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{acos\theta} r^2 drd\theta$	2	K2	CO5
13	Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{acos\theta} r\sqrt{a^2-r^2}drd\theta$.	2	K2	CO5
14	Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yzdxdydz$.	2	K2	CO5
15	Evaluate $\int_0^a \int_0^b \int_0^c (x^2+y^2+z^2)\, dxdydz$.	2	K2	CO5

Part - C / 10 Marks				
Sl. No.	Questions	Marks Split-up	K – Level	CO
1	Evaluate by changing the order of integration $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.	10	K3	CO5

2	Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ changing the order of integration.	10	K3	CO5
3	Change the order of integration and evaluate $\int_0^b \int_0^{\frac{a}{b}(b-y)} dydx$	10	K3	CO5
4	Change the order of integration and evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydydx$	10	K3	CO5
5	Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xydydx$ and hence evaluate it.	10	K3	CO5
6	Change the order of integration in $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xydxdy$ and then evaluate it	10	K3	CO5
7	Evaluate $\iint r^3 drd\theta$, where A is the area bounded between the circles $r = 2sin\theta$ and $r = 4sin\ \theta$.	10	K3	CO5
8	Evaluate $\iint_A r^3 drd\theta$, where A is the area between the circles $r = 2cos\ \theta$ and $r = 4cos\ \theta$.	10	K3	CO5
9	Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing into polar coordinates and hence evaluate $\int_0^\infty e^{-x^2} dx$.	10	K3	CO5
10	By changing into polar coordinates, evaluate the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2} dydx$.	10	K3	CO5
11	By changing into polar coordinates, evaluate the integral $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2) dydx$.	10	K3	CO5
12	Evaluate by changing into polar co-ordinates the integra $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$	10	K3	CO5
13	Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using double integration.	10	K3	CO5
14	Find the area bounded by the curves $y = x^2$ and $x + y = 2$	10	K3	CO5
15	Find using double integration the area of the cardioid $r = a(1 + cos\theta)$	10	K3	CO5
16	Find the area of a circle of radius a by double integration.	10	K3	CO5
17	Evaluate $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$	10	K3	CO5
18	Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$.	10	K3	CO5
19	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dzdydx}{\sqrt{a^2-x^2-y^2-z^2}}$.	10	K3	CO5
20	Find by triple integral the volume of tetrahedron bounded by the planes by $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	10	K3	CO5
21	Find the volume of that portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant.	10	K3	CO5
22	Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals.	10	K3	CO5
23	Evaluate $\iiint_V \frac{dzdydx}{(x+y+z+1)^3}$ where V is the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$.	10	K3	CO5

24	Evaluate $\iiint_V xyz\,dxdydz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$ by transforming into spherical coordinates.	10	K3	CO5
25	By transforming into cylindrical coordinates, evaluate the integral $\iiint (x^2+y^2+z^2)\,dxdydz$ taken over the region of space defined by $x^2+y^2 \leq 1$ and $0 \leq z \leq 1$.	10	K3	CO5
26	Change to spherical polar coordinates and hence evaluate $\iiint_V \frac{dxdydz}{x^2+y^2+z^2}$ where V is the volume of the sphere $x^2+y^2+z^2=a^2$.	10	K3	CO5
27	Evaluate $\iiint \sqrt{1-x^2-y^2-z^2}\,dxdydz$, taken throughout the volume of the sphere $x^2+y^2+z^2=1$ by transforming to spherical polar coordinates.	10	K3	CO5